Hyperbola

1. Prove that the point $P\left[\frac{a}{2}\left(p+\frac{1}{p}\right),\frac{b}{2}\left(p-\frac{1}{p}\right)\right]$ lies on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

   Another point on this hyperbola is given by $Q\left[\frac{a}{2}\left(q+\frac{1}{q}\right),\frac{b}{2}\left(q-\frac{1}{q}\right)\right]$.

   Find the equation of chord PQ.
   Deduce that the equation of the tangent to the hyperbola at P is given by
   $bx(p^2+1)-ay(p^2-1)=2abp$.
   This tangent intersects the x-axis at the point A and the y-axis at the point B.
   Find the area of ΔOAB in terms of p.

2. Given four points $A(a,0), A'(-a,0), B(b,0), B'(-b,0)$, $a \neq b$. A point P moves so that its distances are related by the equation: $AP \cdot AP' = BP \cdot BP'$.

Show that the locus of P is a hyperbola and find the equations of its asymptotes.

3. **(a)** A curve of the form $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$ has asymptotes $y^2=m^2x^2$ and passes through the point $(a,0)$. Find the equation of the this curve in terms of x, y, a and m.

(b) A point P on this curve is equidistant from one of its asymptotes and the x-axis.

Prove that, for all values m, P lies on the curve : $(x^2-y^2)^2=4x^2(x^2-a^2)$

4. The tangents to the hyperbola $b^2x^2-a^2y^2=a^2b^2$ at points A and B on the curve meet at point T.

If the mid-point of AB is M, prove that TM passes through the center of the hyperbola.

Prove that the product of the slopes of AB and TM is a constant.